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Dial M for flavor symmetry breaking

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Abstract: We introduce a realization of Seiberg duality in MQCD that does not involve moving branes. Using related arguments in M-theory and IIA, we investigate the flavor symmetry breaking via flavored magnetic monopole condensation of hep-th/0005076. We verify our results by considering visualizations of M5-branes corresponding to $\mathcal{N} = 2$ vacua which survive soft breaking by an adjoint mass term.

Keywords: Duality in Gauge Field Theories, M-Theory, D-branes, Brane Dynamics in Gauge Theories.

Contents

1. Introduction

Four years ago we learned that superQCD is contained in the low energy description of certain configurations of M5-branes in multi-centered Taub-NUT space [1]. As a result, brane configurations in M theory are able to provide a concrete realization of otherwise mysterious quantum phenomena, such as confinement via magnetic monopole condensation [2, 3, 4] and Douglas-Shenker strings [5, 6]. This approach is known as MQCD and is reviewed in [7].

In this paper, we introduce a novel MQCD description of Seiberg duality in $\mathcal{N}=2$ superQCD softly broken to $\mathcal{N}=1$ by adding a mass term for the adjoint chiral multiplets. We then proceed to construct the M-theory realization of the $\mathrm{U}(N_f) \to \mathrm{U}(r)\times\mathrm{U}(N_f-r)$ flavor symmetry breaking mechanism in $\mathcal{N}=1$ $\mathrm{SU}(N_c)$ gauge theories studied in refs. [8, 9, 10].

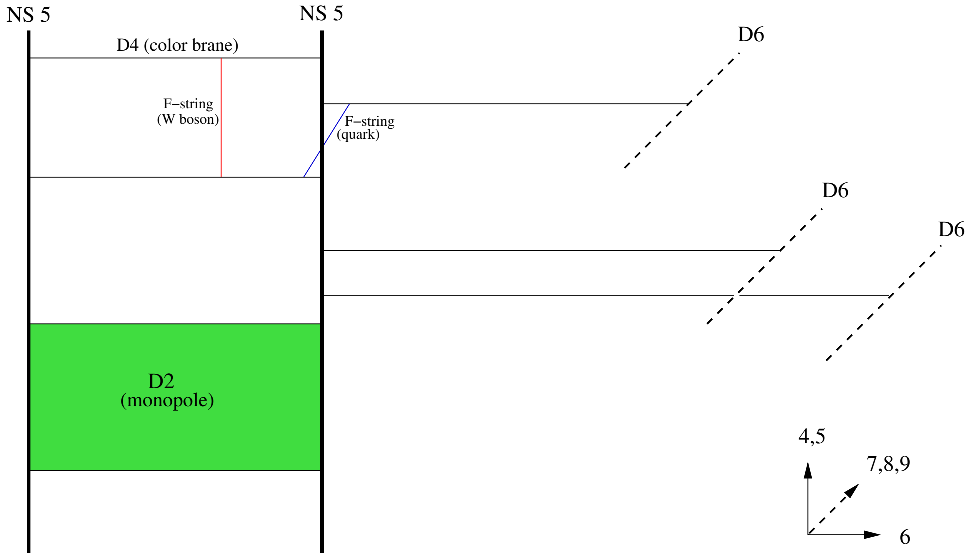

Figure 1: Type IIA string theory realization of $\mathcal{N} = 2$ SU(4) SQCD with 3 flavors. The effective worldvolume of the gauge theory (which lives on the D4 color branes) is spanned by the x^0, x^1, x^2, and x^3 directions.

In section 2, we will review the semi-classical description of supersymmetric gauge theories as effective field theories of parallel D4-branes suspended between NS5-branes in type IIA string theory. To understand the quantum gauge theories we lift this description to M-theory. We then review the field theory results of refs. [8, 9, 10] in section 3. Then, in section 4 we describe Seiberg duality as a choice of two D-brane configurations in IIA which lift to the same M5-brane configuration as seen by M2-brane probes at different energy scales. We finally turn to the MQCD description of flavor symmetry breaking in section 5. A detailed example of the M5-brane configurations corresponding to r-vacua in SU(3) with 4 flavors is presented in the appendix.

2. A review of MQCD

We begin with a review of the standard embedding of 3 + 1 dimensional $SU(N_c)$ SQCD in IIA string theory and its M-theory lift.

2.1 Classical $\mathcal{N} = 2$ SQCD and IIA

Consider type IIA string theory on $\mathbb{R}^{9,1}$ with coordinates $x^0, \ldots, x^9$ and complex coordinates

$$v = x^4 + ix^5\,, \qquad w = x^8 + ix^9\,. \tag{2.1}$$

A stack of N_c parallel D4 "color" branes extend along directions x^0, x^1, x^2, x^3, and x^6. The low energy theory on these branes is SU(N_c) 4+1 dimensional super Yang-Mills with 16 supercharges. We can Kaluza-Klein reduce this to the desired 3+1 dimensional SYM with 8 supercharges by adding two parallel NS5-branes extended along the x^0, x^1, x^2, x^3, x^4, and x^5 directions and placed at positions 0 and L_6 along the x^6 direction with the color branes suspended between them.

The effective gauge coupling, g, of the 3+1 dimensional theory is given by

$$\frac{1}{g^2} = \frac{L_6}{g_s l_s}\,, \tag{2.2}$$

where g_s, l_s, and L_6 are the string coupling constant, the string length and the distance between the two NS5-branes. To decouple the degrees of freedom of the bulk from the color branes we take the limits

$$g_s \;\to\; 0\,, \qquad \frac{L_6}{l_s} \;\to\; 0\,, \qquad g = \text{constant}\,. \tag{2.3}$$

The light perturbative degrees of freedom of the gauge theory are strings which stretch between the color branes, yielding an $\mathcal{N}=2$ vector multiplet transforming in the adjoint representation of $\mathrm{SU}(N_c)$. The distances between color branes correspond to the vacuum expectation values of the adjoint scalars in the vector multiplet and so parameterize the Coulomb branch.

Quark hypermultiplets transforming in the fundamental representation of $\mathrm{SU}(N_c)$ may be included by attaching N_f D4 "flavor" branes stretching between one of the NS5-branes and a D6 flavor brane. The boundary conditions imposed by the D6-brane suppress the five dimensional physics resulting from strings stretched between flavor branes as seen in ref. [21], where additional NS5-branes are considered in place of the D6 flavor branes. To preserve $\mathcal{N}=2$ supersymmetry no two D4-branes may connect the same NS5 and D6-brane. This is known as the s-rule [11] and is U-dual [12, 13] to Pauli's exclusion principle. The quarks and squarks are strings which stretch from one D4 flavor brane to one D4 color brane and so they transform in the fundamental representations of both the $\mathrm{SU}(N_c)$ gauge group and the global flavor symmetry group. Semiclassical magnetic monopole and dyon [1, 14, 15] states are realized by D2-branes with the topology of a disk bounded by a D4-NS5-D4-NS5 cycle. The brane configuration is summarized in table 1.

There is an unbroken global $\mathrm{U}(N_f)$ symmetry when the flavor branes are placed at the same v coordinate, although this symmetry is broken to $\mathrm{U}(1)^{N_f}$ when they are placed at distinct positions $v = m_i$, $i = 1,\ldots,N_f$. The m_i are the bare quark masses. Generally, a quark with flavor i and color $a = 1,\ldots,N_c$ has mass

$$m_i^a = |m_i - \phi^a| \tag{2.4}$$

which is the shortest distance between color brane a and flavor brane i. Alternately the quark mass can be read from the superpotential terms:

$$W \supset \int d^2\theta\ \{\tilde{Q}_i \Phi Q^i + m_i \tilde{Q}_i Q^i\}\,, \tag{2.5}$$

Brane	x^0	x^1	x^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9
D4	X	X	X	X			X			
NS5	X	X	X	X	X	X				
$\mathrm{NS5}_\theta$	X	X	X	X	(X)	(X)			(X)	(X)
D6	X	X	X	X				X	X	X
D2	X				(X)	(X)	X			
D0	(X)						(X)			

Table 1: Alignments of branes in IIA. Parentheses indicate that the brane may be aligned at an angle between the given directions. For the D0-branes they are used to indicate the presence of both D-instantons and dynamical D0 particles.

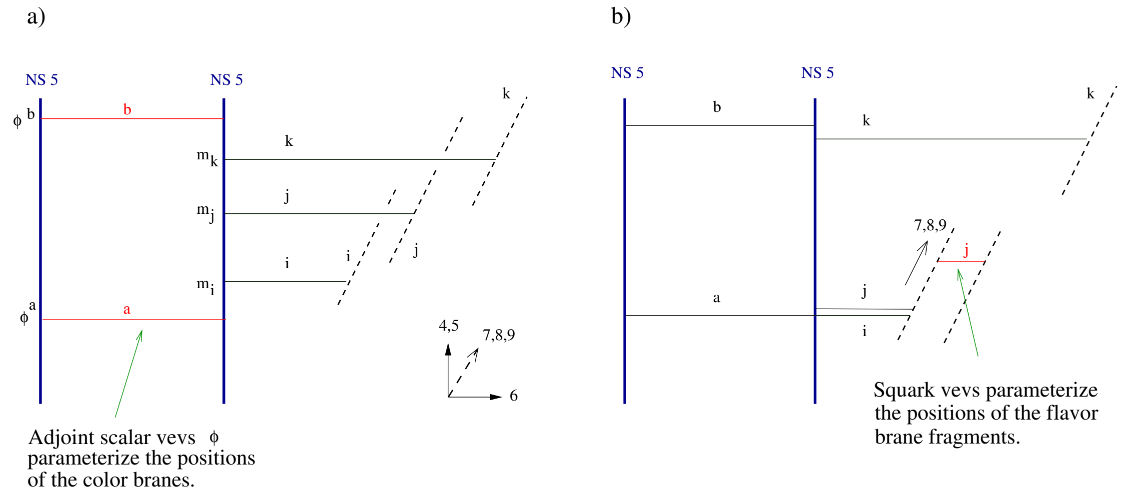

Figure 2: Type IIA brane configurations corresponding to the a) Coulomb branch b) a mixed Coulomb and Higgs branch.

where $Q_i, \tilde{Q}_i$ are the $\mathcal{N} = 1$ chiral multiplets of the quark hypermultiplet and Φ is the $\mathcal{N} = 1$ chiral multiplet of the $\mathcal{N} = 2$ vector multiplet.

When two flavor branes i and j and a color brane a are at the same v coordinate, $\phi^a = m_i = m_j$, then it is possible to enter the Higgs branch of the gauge theory. This is done by connecting color brane a to flavor brane i and then breaking flavor brane j on D6-brane i. At this point we are allowed to move the portion of D4-brane j which is between D6-branes i and j, corresponding to generating vacuum expectation values for the squarks in the hypermultiplet. These vacuum expectation values are parameterized by the position of the D4-brane in the x^7, x^8, and x^9 directions as well as the Wilson line of the gauge field A_6.

The $\mathrm{U}(1)_R \times \mathrm{SU}(2)_R$ R-symmetry of the classical $\mathcal{N} = 2$ theory is manifested as a rotational symmetry of the brane cartoon. The $\mathrm{U}(1)_R$ symmetry corresponds to rotations of the v-plane, while the $\mathrm{SU}(2)_R$ is the universal cover of the SO(3) acting on x^7, x^8, and x^9 by rotations.

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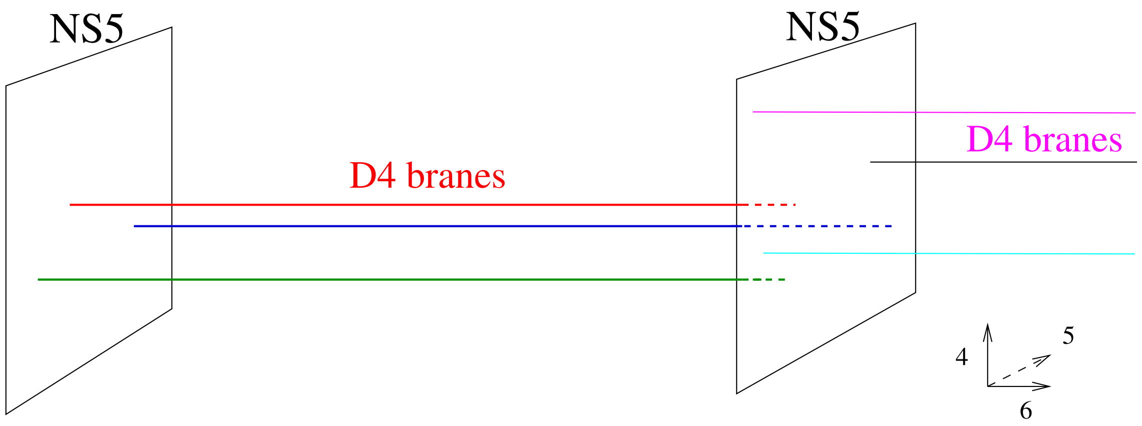

Figure 3: IIA realization of the Coulomb branch of $\mathcal{N} = 2$ SU(3) SQCD with 3 flavors.

2.2 Quantum gauge theory and M-theory

By lifting the above brane configuration to M-theory [1], we can consider non-perturbative (in g_s) quantum corrections and thereby gain considerable insight into the origin of quantum phenomena in SQCD. In particular, we consider the above IIA brane configuration in the limit of large g_s and L_6 with $1/g^2 = L_6/R$ fixed, where the classical description of M-theory is valid.

In M-theory, the D4-brane is an M5-brane which wraps the M-theory circle ($x^{10} \sim x^{10} + 2\pi$) once while an NS5-brane is an M5 brane which does not wrap the x^{10} direction. The collection of D4 and NS5-branes can therefore be described as a single M5-brane $\mathbb{R}^{3,1} \times \Sigma$ which fills the x^0, x^1, x^2, x^3 space and is a Riemann surface Σ in the x^4, x^5, x^6, and x^{10} directions.

If we introduce the new holomorphic coordinate,

$$t = \exp\left(\frac{-x^6}{R} - ix^{10}\right), \tag{2.6}$$

where $R = g_s l_s$ is the radius of the M-theory circle, we can construct Σ explicitly as the vanishing locus of a polynomial $F(v,t)$,

$$F(v,t) = t^2 + t\prod_{a=1}^{N_c}(v - \phi^a) + \Lambda^{2N_c - N_f}\prod_{i=1}^{N_f}(v - m_i) = 0\,, \tag{2.7}$$

where Λ is the dynamically generated QCD scale of the theory. Note that Σ is precisely the Seiberg-Witten curve [2] of the gauge theory. For example, the lift of the brane configuration shown in figure 3 is the single M5-brane pictured in figure 4.

We can immediately see the running of the gauge coupling from the form of the curve. Since the couplings are functions of the mass scales of the lightest charged states in the theory, and as these scales just correspond to distances in v [1], we find that

$$\frac{1}{g(v)^2} = \frac{L_6(v)}{g_s l_s} \;\sim\; \log|v|\,. \tag{2.8}$$

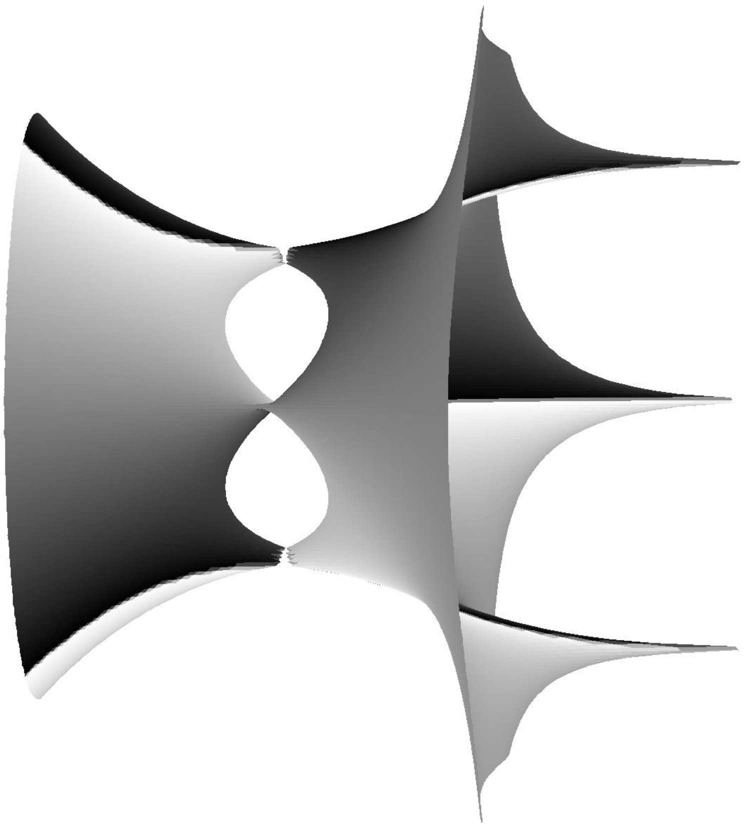

Figure 4: The M-theory lift of the above IIA configuration to a single M5-brane. The directions x^4, x^5, and x^6 are shown explicitly while the x^{10} coordinate is parameterized by darkness.

Also, the $\mathrm{U}(1)_R$ anomaly is particularly easy to see in the M-theory picture. Recall that D4-branes wrap the M-theory direction and end on the NS5-branes, which do not wrap the M-theory direction. This means that the end of a D4-brane on an NS5-brane is a vortex in the embedding coordinate of the NS5-brane in the M-theory direction (see figures 4 and 7). In particular, at large v if one follows a circle along the NS5-brane whose interior contains all of the D4-branes, this circle will wrap the M-theory direction as many times as there are colors minus flavors attached to this NS5-brane. Thus the naive $\mathrm{U}(1)_R$ rotational R-symmetry of the brane must be combined with a simultaneous rotation of the M-theory circle. Such a redefinition is not possible with both NS5-branes as the x^{10} redefinitions would have to be in opposite directions for the two branes and so the $\mathrm{U}(1)_R$ is anomalous. There is a residual $\mathbb{Z}_{2N_c-N_f}$ symmetry in the $\mathrm{U}(1)$ redefined to include a rotation of the M-theory circle in opposite directions on both NS5-branes. For these special angles the two redefinitions only disagree by a multiple of 2π in the M-theory direction.

The reader may verify these claims visually by considering the asymptotic x^{10} dependence of the regions corresponding to the NS5 branes (large positive and negative values of x^6) on the M5-branes pictured in figures 4 and 7.

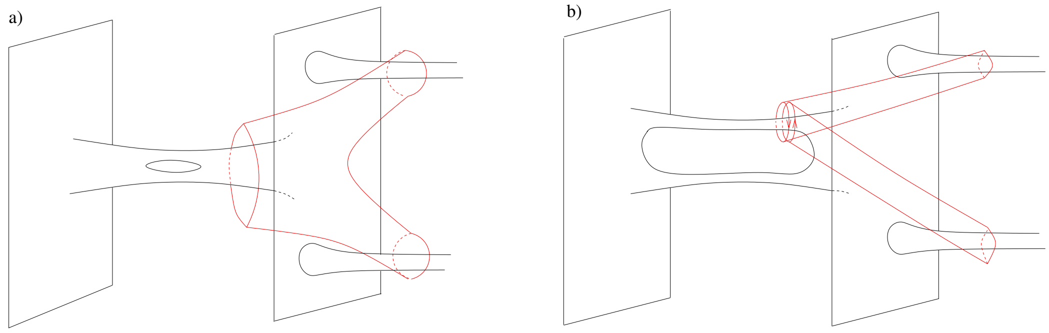

Figure 5: a) a baryon which wraps 2 color and 2 flavor branes in SU(2) SQCD with 2 flavors. b) a meson which wraps 2 flavor branes and wraps and unwraps one color brane.

Another benefit of the M-theory lift is that all the matter in the gauge theory is realized in M-theory by open M2-branes ending on the M5-brane. In reducing to IIA, the M2-branes which wrap the M-theory circle carry fundamental string charge, while unwrapped M2-branes are D2-branes. The M2-brane configurations corresponding to BPS states were analyzed critically in [14, 15]. They found that BPS states were M2-branes with the minimal possible areas in their homology classes.

We can also gain intuition for baryons and mesons in SQCD using M2 branes. For example, baryons are M2-branes [4] with $k+1$ boundaries as seen in figure 5. One boundary wraps k color branes and the other k boundaries each wrap a flavor brane. In the IIA limit, the baryon is a collection of k quarks attached by a k-string [3, 6]. Mesons are tubular M2-branes which start by wrapping one flavor brane, connect to a color brane, and then extend in the space directions (x^1, x^2, and x^3) before wrapping another flavor brane.

We certainly have gained a great deal in this lift, but we don't quite get it for free. Since this lift involves going to large string coupling, one cannot be sure that the low energy effective theory on the M5-brane is the gauge theory we started with. In fact, it is actually a six dimensional theory. The M-theory limit is exactly the opposite limit needed to obtain the 4D gauge theory. It requires taking the radius of the M-theory circle $R = g_s l_s$ as well as L_6 large leaving $\frac{1}{g^2} = \frac{L_6}{R}$ fixed. However, since unbroken supersymmetries protect certain holomorphic quantities (like the masses of BPS states and superpotentials) from perturbative g_s quantum corrections, one can still use classical computations involving the M5-brane to determine them exactly. The breakdown of the M-theory picture for the computation of non-holomorphic quantities (like the masses of non-BPS states) can then be understood as due to the presence of KK modes (dynamical D0-branes and A_6 fluctuations) which become light and strongly coupled in this limit [3].

For example, one can solve the $\mathcal{N} = 2$ gauge theories constructed above at generic points in their moduli spaces as their effective actions are governed by holomorphic

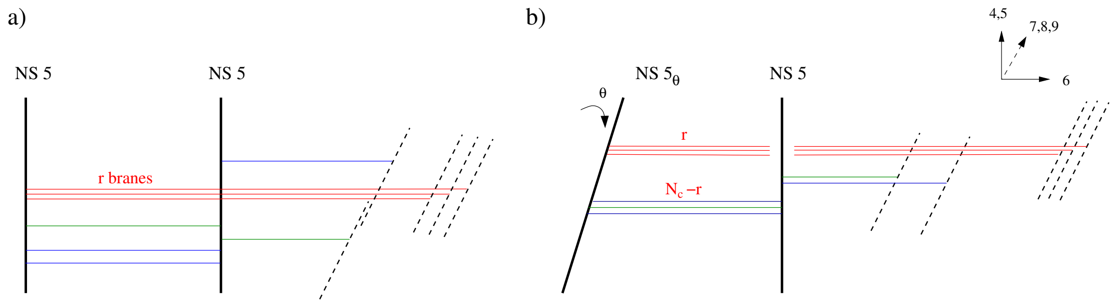

Figure 6: a) r color branes and flavor brane connect. b) After rotating the five brane, color branes which are not connected to flavor branes move to the origin of v. This is the realization of the r vacua in [17].

quantities ($\mathcal{N} = 2$ prepotentials $\mathcal{F}$) which can be computed exactly using Σ [1]. Upon breaking to $\mathcal{N} = 1$, the Kähler potential is no longer protected from such corrections. However, as the superpotential of the $\mathcal{N} = 1$ theory is still protected, we will see that we can still learn what we need about $\mathcal{N} = 1$ SQCD from M-theory.

2.3 Soft breaking to $\mathcal{N} = 1$ SQCD

Rotating one of the NS5-branes

$$\begin{pmatrix} v' \\ w' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} v \\ w \end{pmatrix} \tag{2.9}$$

results in a brane configuration which preserves only four supercharges [16] and softly breaks $\mathcal{N} = 2$ supersymmetry to $\mathcal{N} = 1$. We will refer to the rotated NS5 brane as the NS5$_\theta$-brane. The NS5$_{\pi/2}$-brane is commonly referred to as the NS5$'$-brane in the literature.

The symmetry breaking classical superpotential generated by this process can be understood as follows. After the rotation, color branes at generic positions in v no longer minimize their lengths. In fact, to reach equilibrium, all color branes much either slide to $v \sim O(\Lambda)$ or attach to flavor branes. Since translations along v (which correspond to adjoint scalar vevs) now cause the color branes to stretch, the $\mathcal{N} = 1$ chiral multiplets containing these adjoint scalars acquire a mass μ via the superpotential term

$$W \supset \mu\, \mathrm{Tr}\Phi^2\,, \qquad \mu \;\sim\; \tan\theta\,. \tag{2.10}$$

If r color branes connect to flavor branes, classically the only surviving vacuum is

$$\phi^i = m_i \quad \text{for } i = 1, \ldots, r \qquad \phi^a = 0 \quad \text{for } a = r+1\,, \ldots N_c\,. \tag{2.11}$$

However, when quantum corrections are considered the surviving vacua are those where $N_c - r$ or $N_c - r - 1$ monopoles and dyons become massless and condense with

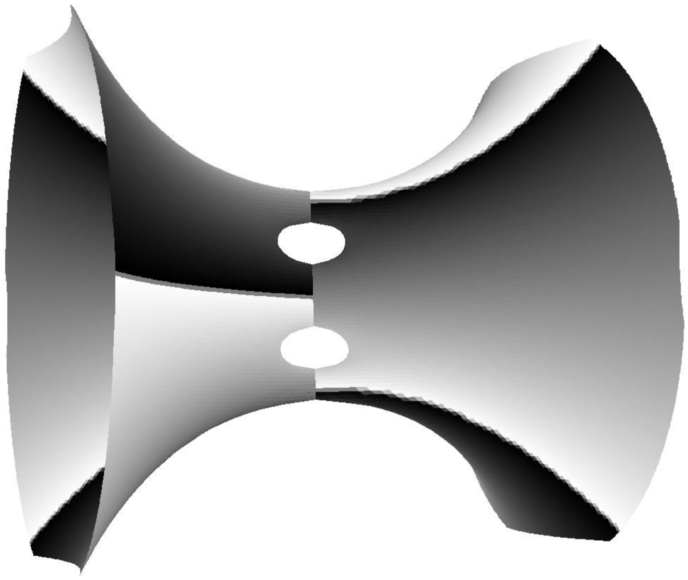

Figure 7: An M5-brane configuration corresponding to $\mathcal{N} = 2$ SU(3) SYM near a vacuum which survives upon softly breaking to $\mathcal{N} = 1$. The two small holes are the degenerating cycles. The M2-branes corresponding to nearly massless monopoles are disks bounded by these cycles. The M theory direction is parameterized by darkness.

vevs of order $O(\mu\Lambda)$. In the M theory picture, this occurs when the bounding cycles of the corresponding M2-branes degenerate, as in figure 7.

More precisely, the vacua which survive the $\mu\mathrm{Tr}\Phi^2$ perturbation are those in which the cycles that have degenerated result in an M5-brane configuration $\Sigma \times \mathbb{R}^4$ such that Σ is of genus zero, as argued in refs. [2, 3, 18].

To preserve SUSY the flavor branes must continue to extend along the x^0, x^1, x^2, x^3, and x^6 directions, in particular they cannot rotate into the w directions. Therefore when the NS5_θ-brane rotates, the flavor branes ending on it must translate in w, sliding along the corresponding D6-brane. This translation corresponds to meson vevs [7, 18].

3. Summary of field theory results

The dynamics of the $\mathcal{N} = 2$ supersymmetric SU(N_c) gauge theories constructed above and the dynamical breaking of flavor symmetry have been studied in detail in refs. [8, 9, 10]. Here we briefly summarize the results.

The theory with N_f massless quark hypermultiplets has U(N_f) flavor symmetry, $\mathrm{SU}(2)_R$ symmetry, and a non-anomalous discrete $\mathbb{Z}_{2N_c-N_f}$ subgroup of $\mathrm{U}(1)_R$. We are interested in the $\mathcal{N} = 1$ perturbation of the theory by the adjoint mass term μ Tr Φ^2. The moduli space contracts to the set of points that give maximally degenerate (genus zero) Seiberg-Witten curves.

In the semi-classical regime with a large adjoint VEV, there are 't Hooft-Polyakov magnetic monopoles. Zero modes of the quarks around a monopole generate flavor quantum numbers for the magnetic monopoles. It was shown that they come in completely anti-symmetric rank-r tensor representations with ${}_{N_f}C_r$ multiplicities.

The strongly coupled regime was studied with a variety of techniques. When $N_f < N_c$, there are vacua parameterized by an integer $r = 0, 1, \cdots, [N_f/2]$ ($[x]$ is the Gauss' symbol) where the flavor symmetry is broken dynamically as $\mathrm{U}(N_f) \to \mathrm{U}(r) \times \mathrm{U}(N_f - r)$. If $r < N_f/2$, the physics around the vacuum can be described by an IR free effective lagrangian [17] (a "magnetic dual" to the asymptotically free semiclassical $\mathrm{SU}(N_c)$ description) with $\mathrm{SU}(r) \times \mathrm{U}(1)^{N_c-r-1}$ gauge group. N_f "magnetic quark" hypermultiplets transform as the fundamental representation of $\mathrm{SU}(r)$ while there are "magnetic monopoles" for each of the "magnetic" $\mathrm{U}(1)$ factors. When perturbed by the adjoint mass term, all gauge groups are Higgsed by the condensates of magnetic objects, corresponding to the confinement of the electric theory. It was argued in ref. [8, 9] that the semi-classical monopoles in the rank-r anti-symmetric tensor representation smoothly match to the baryonic composites of magnetic quarks of the low-energy $\mathrm{SU}(r)$ theory based on circumstantial evidence. Upon mass perturbations, one can count the number of vacua:

$$\mathcal{N}_1 = (2N_c - N_f)2^{N_f-1}\,, \tag{3.1}$$

originating from r-vacua ($r \leq [N_f/2]$) with $(2N_c - N_f)$ copies due to the $\mathbb{Z}_{2N_c-N_f}$ symmetry. Therefore *the flavor symmetry breaking and confinement have a common origin in these theories*: condensation of magnetic objects with non-trivial flavor quantum numbers. Strictly speaking, however, the existence of monopoles in the anti-symmetric tensor representations was demonstrated only in the semi-classical regime and its extrapolation to the strongly coupled regime and the matching to the baryonic composite was a conjecture. When $r = N_f/2$ (possible obviously only when N_f is even), the low-energy magnetic gauge group is superconformal with an infinitely strong coupling $\tau = -1$. Due to some reason, the same low-energy effective action seems to describe the dynamics of flavor symmetry breaking even though there is no weakly coupled description of the theory.

When $N_f > N_c$, there is a new vacuum without flavor symmetry breaking. It is at the same point on the moduli space as the $r = N_f - N_c$ vacuum, while the finite quark mass perturbation shows that there are additional

$$\mathcal{N}_2 = \sum_{r=0}^{N_f-N_c-1} (N_f - N_c - r)\,{}_{N_f}C_r \tag{3.2}$$

vacua with unbroken flavor symmetry.

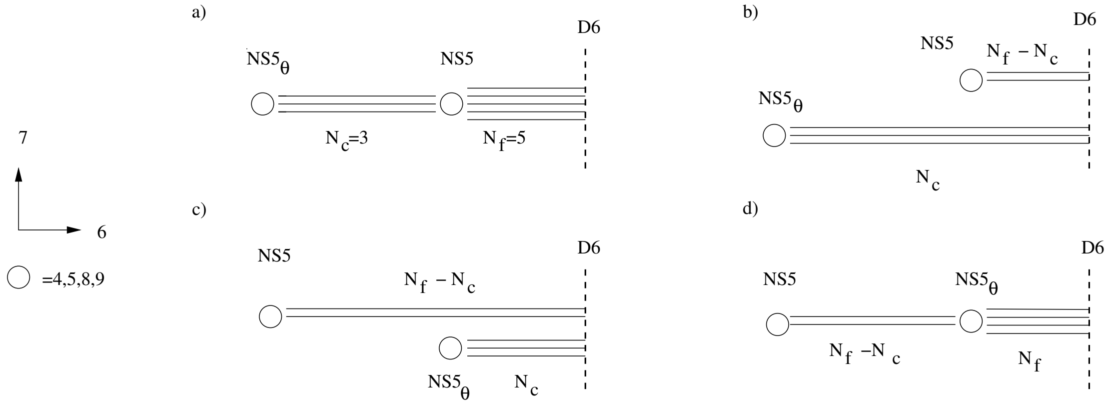

Figure 8: In IIA Seiberg duality is realized by brane exchange.

4. Seiberg duality from M-theory

As we noted in the last section, the effective theories at the r-vacua which survive SUSY breaking are IR free theories for $r < N_f/2$. In particular, at the baryonic root we find a weakly coupled theory whose low energy physics is well described by an IR free SU($\tilde{N}_c$) gauge theory, where $\tilde{N}_c = N_f - N_c$. This is very reminiscent of Seiberg duality in $\mathcal{N} = 1$ theories [19]. In [17], this observation was used to "derive" $\mathcal{N} = 1$ Seiberg duality by mass perturbing the $\mathcal{N} = 2$ theory.[1] We will consider this scenario using M-theory.

Seiberg duality was first realized in IIA string theory in ref. [20] via the exchange of two NS5-branes, as illustrated in figure 8. To avoid a singular configuration the authors first displaced one of the NS5-branes in the 7 direction, which corresponds to turning on a Fayet-Illipolous term in the field theory description. This process corresponds to Seiberg duality with one caveat, the full U(N_f) × U(N_f) flavor symmetry is not realized in the above IIA configuration nor in its M-theory lift.

An alternate proposal for realizing Seiberg duality via string/M theory was made later that year by Schmaltz and Sundrum [21]. Using the M-theory lift of the above brane setup, they found that Seiberg duality could be understood by taking the $\Lambda \to 0$ and $\Lambda \to \infty$ limits of the resulting single M5 brane configuration. In particular, they found that the two limits were related by an exchange of branes and therefore correspond to an electric theory and its magnetic dual. However, their arguments depended on turning on finite bare quark masses.

The following year this scenario was clarified and extended to the massless case by Hori [22]. He observed that in M-theory, the M5-branes can be crossed with no singularity even without the Fayet-Illiopolous parameter. In fact, at the root of the baryonic branch, when $N_f \geq N_c + 1$ the M5-brane consists of two connected

[1] Unfortunately, the result was not quite Seiberg duality as there was an extra non-renormalizable coupling in the effective action associated with integrating out the massive adjoint scalar which becomes relevant in the low energy limit.

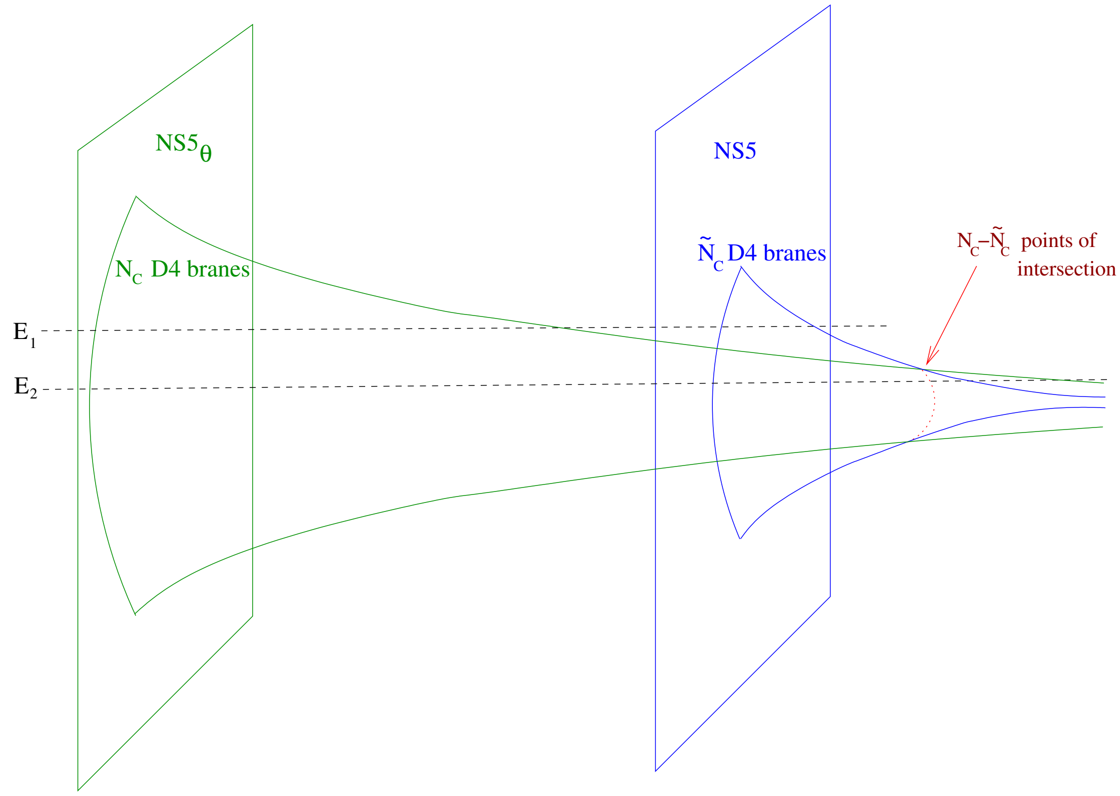

Figure 9: At the root of the baryonic branch, the low energy physics (to the right of the intersection) is the $\mathrm{SU}(\tilde{N}_c) \times \mathrm{U}(1)^{N_c-\tilde{N}_c}$ magnetic theory.

components [18] and the duality corresponds to simply translating one past the other along x^6. The x^6 coordinate of a connected component does not affect the field theory description and so Hori argues that this process preserves its universality class.

Most recently, in ref. [23] Seiberg duality was understood within the context of geometric engineering as a birational flop in a T dual description consisting of D5-branes which are dual to the D4 branes in IIA and degenerations in the T-dualized circle which are dual to the NS5-branes.

We propose a new description of Seiberg duality which has no moving branes and is valid at any fixed, finite value of Λ. Consider the case $N_f > N_c$ at the root of the baryonic branch in an $\mathcal{N}=2$ theory with no bare quark masses. Then according to ref. [18] the connected components of the M5-brane are described by

$$t = v^{N_c} \qquad \text{and} \qquad t = v^{\tilde{N}_c}, \quad \tilde{N}_c = N_f - N_c \tag{4.1}$$

as seen in figures 9 and 10. These two M5 branes intersect at $N_c - \tilde{N}_c$ points.

The crucial realization is that the reduction of this configuration to IIA and in particular the x^6 location of the NS5-branes is not uniquely defined [1]. Rather, we claim, that the effective location of the NS5-branes depends on the energy scale probed, E. We consider charged hypermultiplet matter corresponding to M2-branes

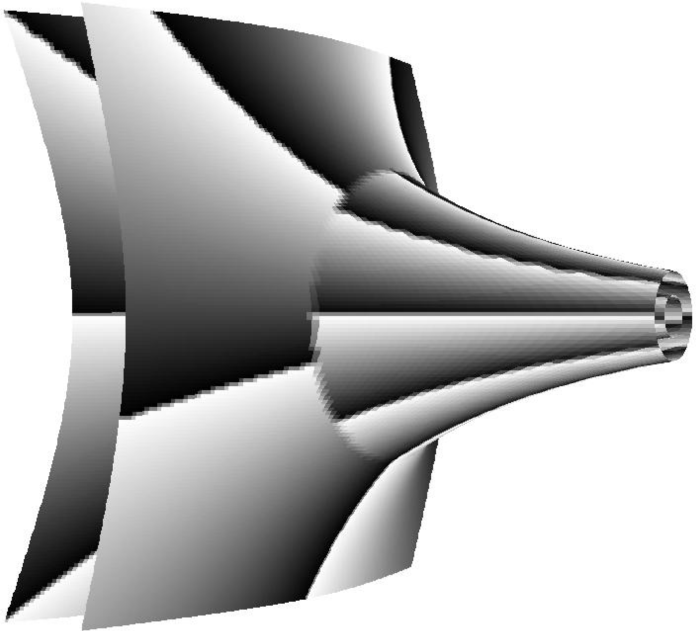

Figure 10: Baryonic root of SU(8) with 13 flavors.

of disk topology stretched between the two branches of the M5-brane. It is clear from figure 10 that the area of such an M2-brane (and therefore its energy) is proportional to the distance in v between the two branches of M5-brane at its position in x^6. This restricts the regions on the M5-brane that a quark of a given energy can probe. Thus, for such a probe, one may effectively place an NS5-brane at the intersection of the corresponding M5-brane and $v \sim E$. A high energy $E_1 \gg \Lambda$ (semiclassical) M2-brane probe is restricted to probe large features at small x^6. Thus, to such a probe, the configuration consists of an NS5-brane with an NS5$_\theta$ brane on its left, N_c color branes connecting them and N_f semi-infinite flavor branes which extend to the right. That is, the reduction to type IIA is figure 8a, which can roughly be obtained by drawing an NS5-brane wherever the line E_1 crosses an M5-brane. This corresponds to the SU(N_c) asymptotically free electric theory. However, low energy quarks can only exist at sufficiently large x^6. Thus, a low energy $E_2 \ll \Lambda$ M2 brane probe corresponding to a charged quark is only sensitive to the configuration of the M5-brane at large x^6, the right side of figure 9 and 10. Thus, if we consider the portion of the M5-brane configuration accessible to such a probe, we would find that the corresponding reduction to IIA at E_2 is figure 8d.

Now, note that the two M5-branes cross at the QCD scale Λ, and at energy scales below this, that is, further right in the figure, the M5-brane is to the left of the M5$_\theta$-brane. Thus a probe at energies below the QCD scale will see the two M5-branes interchanged, which is the usual description of the magnetic theory. This theory is IR free because the branes separate as v increases, and has a Landau pole where the branes cross.

5. Flavor symmetry breaking

5.1 Flavored magnetic monopoles

Recall that a flavorless magnetic monopole (or dyon) in a IIA realization of the electric picture is a D2-brane with disk topology bounded by a circle which extends along one color brane, down an NS5 brane, back along another color brane and then finally back along the other NS5-brane, as in figure 1. More generally magnetic monopoles may be charged under the $\mathrm{U}(N_f)$ flavor symmetry. In this case the monopole may include fundamental strings extending from the D2-brane to a flavor brane. Another version of the s-rule states that at most one such fundamental string can extend from a given monopole to a given flavor brane, which identifies these strings as excitations of the fermionic zeromodes present in flavored monopoles.

The M-theory lift of this monopole configuration is a single M2-brane with the topology of a disk. Thus, its boundary is a circle which wraps the M-theory direction a total of zero times, as seen in figure 11.

The microscopic mechanism behind the flavor symmetry breaking of refs. [8, 9, 10] is the condensation of magnetic monopoles in an antisymmetric tensor flavor representation drawn in figure 11a. The transformation properties under $\mathrm{U}(N_f)$ can be read off from the brane cartoon. If the monopole does not wrap any flavor branes it transforms as a flavor singlet. A monopole that wraps one of the flavor branes transforms in the fundamental representation, while wrappings of more than one flavor brane transform in the antisymmetric representation of the flavor group. To see why this representation is antisymmetric, notice that in IIA, if the D6-branes are moved between the NS5-branes, a monopole consists of a D2-brane connected by strings to D6-branes. The s-rule provides an exclusion principle, restricting the number of strings connecting a monopole to a D6-brane to 0 or 1. As a consistency check on this picture, notice that there are 2^{N_f} configurations of wrappings which agrees with the known number of states in the representation.

In order to understand how semiclassical magnetic monopoles in the UV theory are related to the IR degrees of freedom, we can consider an M2 brane corresponding to a high energy monopole configuration and follow its decay into the IR. Deforming the UV theory away from the baryonic root to clearly visualize its charges, such a monopole is pictured in figure 11a. We deform back to the baryonic root and then allow it to decay, requiring that its wrappings (i.e. charges) are preserved. Its energy will become sufficiently low that it is best described using the dual magnetic description, that is, with the NS5 branes switched. Now, if we deform the IR theory to a generic point in its Coulomb branch and keep track of the wrappings, this configuration corresponds to magnetic baryons as seen in figure 11b. The magnetic theory is IR free, and so these baryons decay into magnetic quarks whose condensation provides the order parameter of the flavor symmetry breaking.

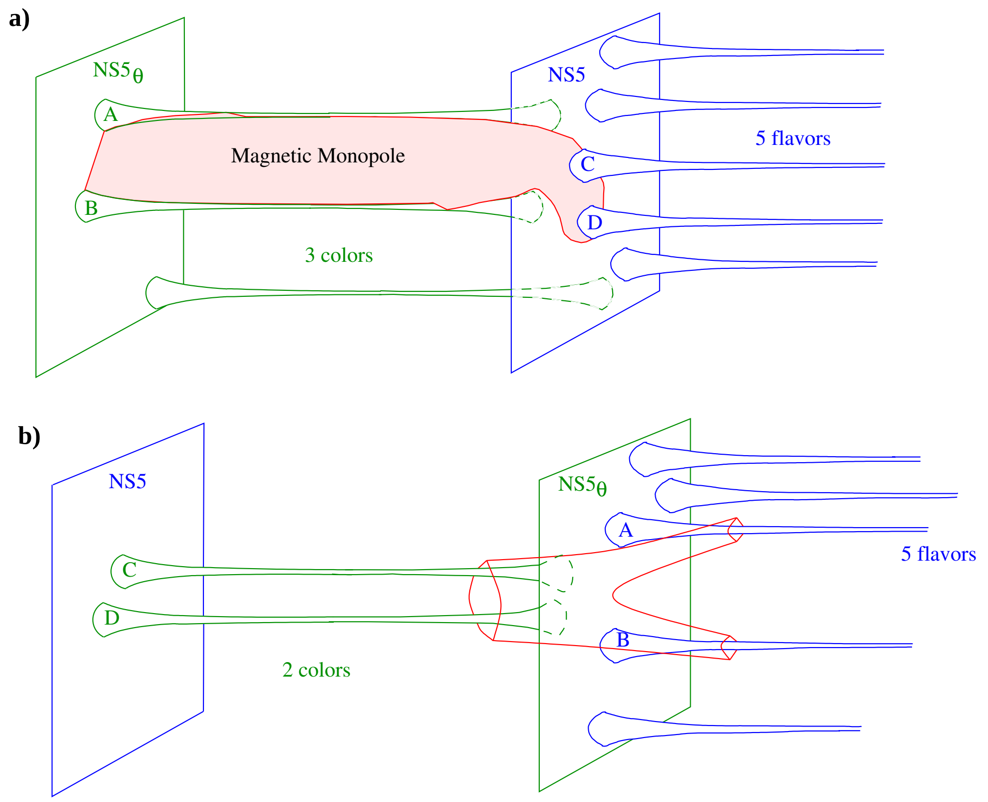

Figure 11: a) A magnetic monopole in an antisymmetric tensor flavor representation. b) In the dual magnetic picture it becomes a baryon.

It is also easy to see that the correlation between the electric charge and chirality of dyons discussed for SU(2) gauge theories in ref.[24] follows from the fact that the monopole is topologically a disk. All monopoles with even chirality come from monopoles whose boundary wraps an even number of flavor branes and therefore the M-theory direction an even number of times along the M5 brane as well. Each flavor brane wrapping introduces a hypermultiplet into the antisymmetric representation and hence the monopole has chirality $(-1)^H = 1$. Each color brane wrapped yields a unit (using the conventions of [24]) of electric charge and so the monopole acquires an even electric charge. This argument works similarly for odd chirality and odd electric charge.

5.2 Symmetry breaking pattern

As we reviewed in section 3, field theory calculations in a variety of limits showed that flavor symmetry is generically broken

$$U(N_f) \to U(r) \times U(N_f - r) \tag{5.1}$$

in softly broken $\mathcal{N} = 1$ asymptotically free ($N_f < 2N_c$) SQCD in the limit that bare quark masses vanish. We will draw string and M-theory realizations of these limits

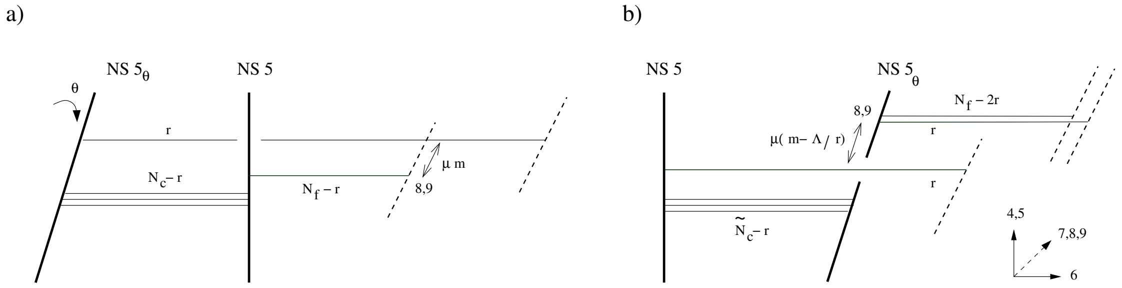

Figure 12: a) Semiclassically when SUSY is broken by rotating the NS5_θ-brane, r color branes connect to flavor branes while the rest slide to v=0. b) The dual magnetic description of the nonbaryonic branches can be understood similarly in the semiclassical case.

and use them to reproduce the qualitative results of several field theory calculations. In particular, in the semiclassical limit, we will equate r with the number of color branes attached to flavor branes.

5.3 Semiclassical analysis

Following refs. [8, 9, 10] we begin by considering bare quark masses much larger than the QCD scale so that a semiclassical (IIA) analysis is valid. This will allow us to count the total number of vacua for comparison with later calculations. Recall from section 2.3 that after rotating the NS5_θ-brane all color branes must either slide to $v = 0$ or connect to a flavor brane as in figure 12a. The number of color branes connecting to flavor branes, r, clearly can neither exceed the number of color branes nor the number of flavor branes. We illustrate the simple case of the vacua arising this way with the bare quark masses all equal $m = m_i \gg \Lambda$ in the appendix for the case of SU(3) with $N_f = 4$.

To count the number of vacua with generic quark masses, notice that r flavor branes can attach to color branes in $\binom{N_f}{r}$ ways (recall that there is no combinatoric factor from choosing which color branes to attach as these choices are gauge equivalent), leaving $N_c - r$ color branes which form a line[2] [6] centered at $v = 0$, as shown in figure 7. This line can be rotated by integer multiples of $\pi/(N_c - r)$ without affecting the x^{10} coordinates asymptotically far away, corresponding to the anomaly-free R-symmetry subgroup. The $N_c - r$ inequivalent orientations of the line result in $N_c - r$ different vacua, in agreement with the Witten index of this theory. Thus the total number of semiclassical vacua (assuming all bare quark masses are distinct and nonvanishing) is

$$\mathcal{N}_{sc} = \sum_{r=0}^{\min(N_c+1, N_f)} (N_c - r) \binom{N_f}{r} \tag{5.2}$$

in agreement with computations from field theory considerations in refs. [8, 9, 10].

[2] Actually they form an ellipse whose semi-minor axis scales with θ. This ellipse degenerates to a line when $\theta = 0$ and a circle with an A_{N_c-r-1} singularity in its center at $\theta = \pi/2$.

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5.4 Nonbaryonic branches

Semiclassically flavor symmetry is broken by meson vevs equal to $\mu m_i \sim \mu \tan\theta$ which is the distance in the w plane shown in figure 12a. These vevs vanish when the bare quark masses vanish, apparently restoring the explicitly broken flavor symmetry. However we will see that, if we include quantum effects, in some vacua the flavor symmetry remains broken even in this limit. For simplicity let all of the bare quark masses be equal $m = m_i \ll \Lambda$.

The $r = N_f/2$ theory is superconformal in the IR and generally difficult to understand. An example of M5-brane associated with such a vacuum is drawn in figure 13. The two branches, corresponding to different NS5-branes upon reduction to IIA, intersect at exactly two points. Cross-sections to the left and right of these two singularities are seen in the first and second lines of figure 14. They are topologically distinct and correspond to distinct reductions to IIA, yielding different effective field theories.

We will be interested in the IR free case of $0 < r < N_f/2$, corresponding to non-

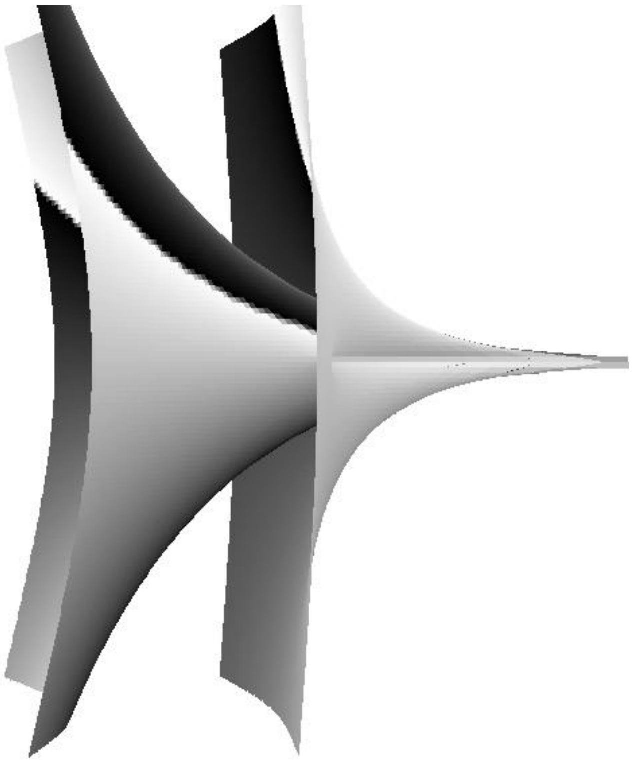

Figure 13: This M5-brane configuration corresponds to the $r = 2$ nonbaryonic root of SU(3) with 4 flavors. In the IR (the right side of the picture) a reduction to IIA produces two parallel, almost coincident NS5-branes indicating that the theory is strongly coupled. The fact that the distance between the branes converges indicates that the IR theory is superconformal.

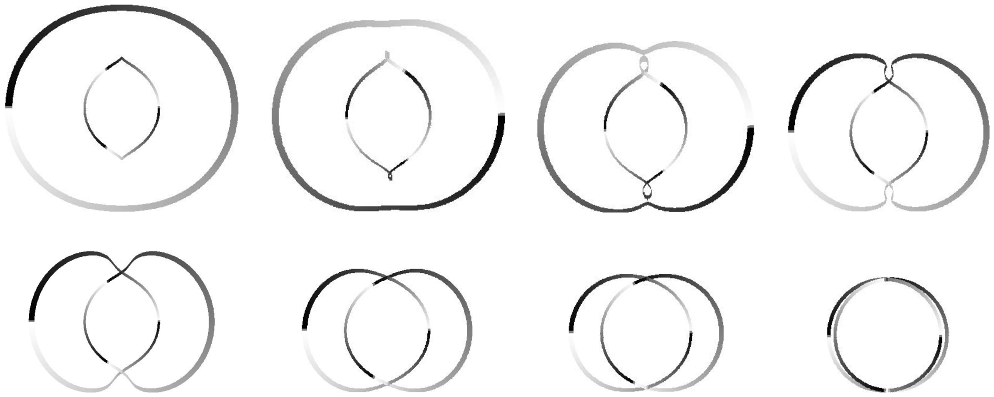

Figure 14: Above are M5-brane cross-sections at constant x^6 corresponding roughly to constant energy scales of the $r = 2$ nonbaryonic root of the SU(3) theory with 4 flavors and no bare quark masses. Reading from left to right and top to bottom the energy scale of the corresponding M2-brane probes decreases. The M theory direction is parameterized by darkness.

baryonic branches. The fundamental degrees of freedom must be magnetic because in the UV this theory has a Landau pole (M5-branes cross) separating it from the semiclassical region with electric degrees of freedom. Thus flavor symmetry breaking can only be caused by magnetic quark vevs, which like meson vevs (meson vevs are quadratic in quark vevs) correspond to distances in the w plane. Semiclassically this distance was μm_i and so vanished when the bare quark masses were taken to zero. Quantum mechanically these flavor branes have width $O(\Lambda)$, critically changing the distances between them. As a result the r nonvanishing magnetic quark vevs

$$q = \sqrt{\mu(m_i - \Lambda/r)} \tag{5.3}$$

do not vanish when the bare masses are eliminated, as seen in figure 12b. These quark vevs break the flavor symmetry

$$U(N_f) \ \rightarrow \ U(r) \times U(N_f - r)\,. \tag{5.4}$$

These vevs also break the global $\mathbb{Z}_{2N_c - N_f}$ symmetry and so there must be $2N_c - N_f$ copies of this configuration. When $r = 0$, $q = \sqrt{\mu m_i}$ and flavor symmetry is unbroken. Again there are $\binom{N_f}{r}$ ways to choose r quarks, leading to a total of

$$\mathcal{N}_1 = (N_c - \tilde{N}_c)2^{N_f - 1} \tag{5.5}$$

vacua of this type. Notice that when $N_f < N_c$, $\mathcal{N}_1$ agrees with $\mathcal{N}_{sc}$ and therefore by supersymmetry this is a complete classification of the vacua.

5.5 Baryonic branch

The above analysis is incomplete when $N_f \geq N_c$ because every color brane can be broken by a D6-brane and the two halves displaced from each other along the w plane

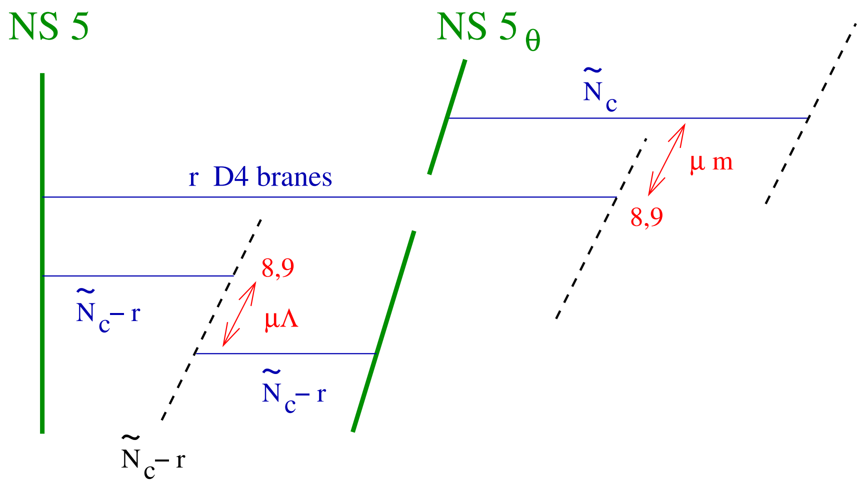

Figure 15: On the baryonic branch, the baryon vev $\mu\Lambda$ is realized as a relative slide of the two halves of the configuration along a collection of D6-branes.

by a distance (baryon vev) exactly canceling the displacement measured by the dual quark vev. Clearly this requires $N_f \geq N_c$ because N_f is the number of D6-branes while N_c is the number of color branes, and each color brane requires a D6-brane along which to break, as illustrated in figure 15.

More concretely, at the root of the baryonic branch the unattached $N_c - \tilde{N}_c$ color branes form a circle. This means that $N_c - \tilde{N}_c$ magnetic monopoles (or dyons), which are between adjacent color branes, become massless and can acquire vevs. The baryonic branch has one more massless monopole compared to the nonbaryonic branch, which is enough to completely Higgs the $\mathrm{U}(1)^{N_c-\tilde{N}_c}$, and therefore there are enough vevs to control the center of mass motion of this set of color branes. The center of mass mode of the entire system is infinitely massive and so a shift in the center of mass of the $N_c - \tilde{N}_c$ branes leads to an opposite center of mass shift of the Seiberg dual $\mathrm{SU}(\tilde{N}_c)$ gauge theory along the w plane, the same shift parametrized by all quark vevs. One result is that the magnetic monopoles are charged under the $\mathrm{U}(1)_B$ baryon number. The crucial implication is that a slide along the w plane in the $\mathrm{U}(1)^{N_c-\tilde{N}_c}$ system can undo the w-shift in the $\mathrm{SU}(\tilde{N}_c)$ configuration which led to the flavor symmetry breaking quark vev. Thus flavor symmetry is unbroken on the baryonic branch.

These geometrical quantities can be related to the corresponding field theory calculation [8, 9, 10] by considering the following superpotential terms [17] in the magnetic description of the baryonic root

$$W \supset \frac{1}{\tilde{N}_c}\mathrm{Tr}(q\tilde{q})\left(\sum_{k=1}^{N_c-\tilde{N}_c}\psi_k\right) - \sum_{k=1}^{N_c-\tilde{N}_c}\psi_k e_k \tilde{e}_k + \mu\Lambda \sum_{k=1}^{N_c-\tilde{N}_c} x_k \psi_k\,. \tag{5.6}$$

Here q and $\tilde{q}$ are magnetic quarks, e_k and $\tilde{e}_k$ are flavorless magnetic monopoles,

x_k are constants and ψ_k are the dual photons of the abelian part of the $\mathrm{SU}(\tilde{N}_c) \times \mathrm{U}(1)^{N_c-\tilde{N}_c}$ dual gauge group. From the superpotential we see that each magnetic monopole is charged under a U(1) while the dual quarks are charged under all of the U(1)'s.[3] As a result the D term equation for each U(1) is

$$\frac{1}{\tilde{N}_c}\mathrm{Tr}(q\tilde{q}) - e_k\tilde{e}_k + \mu\Lambda x_k = 0\,. \tag{5.7}$$

This means that, because there are as many e_k's as x_k's, $\mathrm{Tr}(q\tilde{q})$ can vanish if each $e_k\tilde{e}_k$ is chosen correctly and, in fact, this solution is consistent with the rest of the D and F term constraints. Thus we see that the D term equation for the difference of two vevs is interpreted in M-theory as the following statement. If the connected components of the M5-brane slide apart in the w plane along the D6-branes bisecting the $\tilde{N}_c - r$ color branes then, because the components are rigid, they also separate along the w plane at the semi-infinite flavor branes. In other words by trading a monopole vev for a magnetic quark vev, we find vacua which preserve flavor symmetry. These two distances, whose differences are preserved, are marked with double-headed arrows in figure 15.

To count vacua on this branch, consider the dual $\mathrm{SU}(\tilde{N}_c)$ theory whose gauge symmetry is broken by the adjoint scalar vevs of the r color branes attached to flavor branes. As in the semiclassical case, the remaining $\tilde{N}_c - r$ color branes form a line which can have $\tilde{N}_c - r$ orientations preserving the M theory coordinate asymptotically far away, yielding a multiplicity of $\tilde{N}_c - r$ times the combinatoric factor of $\binom{N_f}{r}$ from the choice of which flavor branes to connect. In all, this provides

$$\mathcal{N}_2 = \sum_{r=0}^{\tilde{N}_c} (\tilde{N}_c - r)\binom{N_f}{r} = \mathcal{N}_{sc} - \mathcal{N}_1 \tag{5.8}$$

states, and thus completes the classification of vacua.

6. Conclusion

We have constructed a new realization of Seiberg duality that relies on an energy scale dependent reduction of M-theory to IIA. We have found the M2-branes that correspond to flavored magnetic monopoles and argued that they correspond to magnetic baryons in the dual magnetic theory, which in turn decay to magnetic quarks. And finally we have interpreted baryon vevs as the relative sliding of two halves of an M theory configuration along a Taub-NUT singularity.

As an application of the above constructions, we have reproduced the field theory results of [9]. In particular we have correctly reproduced the flavor symmetry

[3] This is in contrast to the nonbaryonic root, where there is one less massless magnetic monopole and so after a basis change the dual quarks and magnetic monopoles are charged under disjoint U(1)'s.

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breaking patterns, the order parameters of the symmetry breaking and the counting of states in various regimes. We can interpret these countings in terms of discrete rotations of a line of M5-brane in the v plane.

The case $r = N_f/2$ is difficult to analyze using traditional field theory techniques, as it is superconformal and strongly coupled in the IR. However it is possible that by deforming the corresponding curve, an analysis similar to the Seiberg duality of monopoles above may be possible in this M-theory setting.

A natural next step is to add orientifold planes and try to repeat this analysis for SO(N) and SP(N) gauge theories.

Acknowledgments

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A. An example, SU(3) with $N_f = 4$

We will consider in some detail the M5-branes associated with the r-vacua which survive breaking to $\mathcal{N} = 1$ for the case of SU(3) with $N_f = 4$ and equal quark masses, $m_i = m$. This case is rather special in that the coordinates of these vacua in the moduli space can be analytically determined as a function of mass. Thus, we can explicitly draw out the corresponding M5-brane configurations and test our claims for the brane interpretations of the r vacua in various regimes.

In the case of equal and large bare quark masses, we can clearly see that the M5-brane configurations which survive breaking to $\mathcal{N} = 1$ in figure 16 do indeed correspond to r flavor branes and color branes connected to each other.

In the massless case, the relevant curves are more difficult to interpret, though we include them for completeness in figure 17. Note that in the limit that $m \to 0$, the $r = 1$ non-baryonic root and the baryonic root converge to the same point in moduli space, the $m = 0$ baryonic root. Just as in the superconformal case, it is easier to interpret these brane configuration if we consider their cross-sections in figures 18 and 19, though we will leave the interpretation of these cross-sections for future work.

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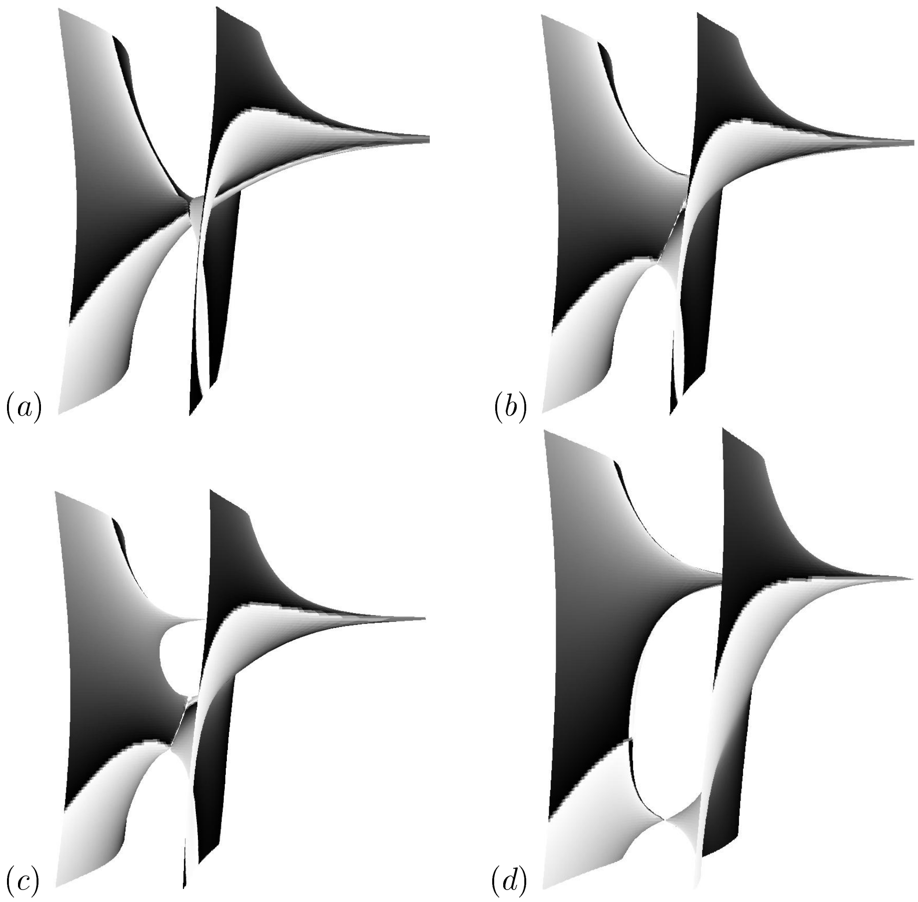

Figure 16: The M5-brane configurations corresponding to r-vacua in SU(3) with 4 flavors in the semiclassical limit, i.e. with $m_i = m \gg \Lambda$. The flavor branes and color branes which have connected are represented by semi-infite tubes extending from the left branch of the M5-brane. The connections between the two NS5-branes are the remaining color branes, pairs of which have condensed massless monopoles between them. (a) The baryonic root. (b) A non-baryonic $r = 0$ root. (c) A non-baryonic $r = 1$ root. (d) The non-baryonic $r = 2$ root.

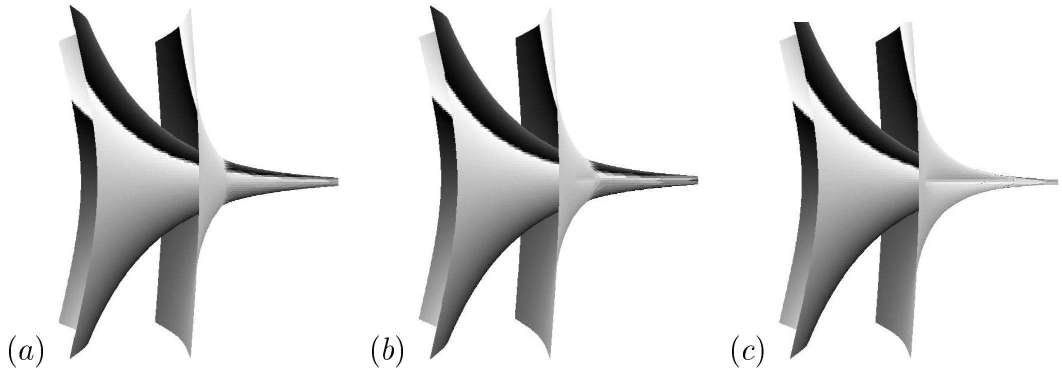

Figure 17: M5-brane configurations at the r vacua in SU(3) with 4 flavors and $\forall m_i = 0$. (a) The baryonic root. (b) An $r = 0$ vacuum. (c) The $r = 2$ vacuum.

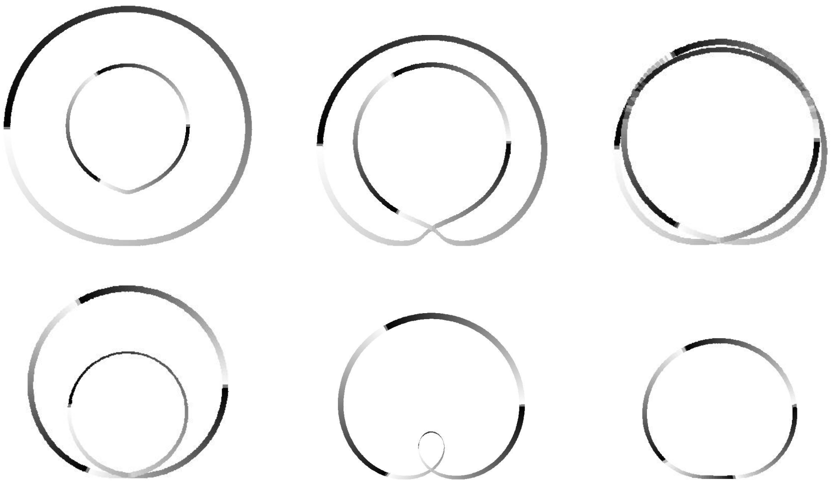

Figure 18: Cross sections at constant x^6 of the M5-brane of figure 17b (the r=0 root).

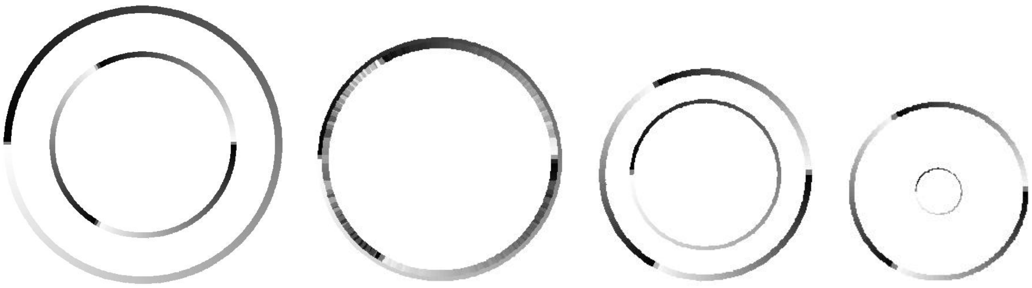

Figure 19: Cross sections at constant x^6 of the M5-brane of figure 17a (the baryonic root).

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